

Game Theory - History and Characters:-

Game theory is a form of competition in which two or more competitors are involved in decision making in anticipation of certain outcomes over a period of time.

Ex - T.V programmes

- In 20th century in 1944 John Von Neumann and Oscar Morgenstern published a book i.e. Theory of games & economic behaviour where they discussed how business of all type may use this techniques to determine the best strategies given in a competitive business environment. The theory of games depends upon following factors.

1. Number of players - (minimum - 2 players) / ∞
2. Sum of gain and losses.
3. Strategies - A set of rules exist (one/many)
4. Timing - both players have to decide simultaneously. means apply their strategies at the same time
5. Conflicting goals:- Each party is interested in maximising his or her welfare at the expense of other.
6. Repeated game :- A one shot decision is termed as play. A series of repetitive decisions is called game
7. Pay off:- The consequences of decision of (termed) the opponents in the game are measured in terms of

certain units (e.g. profit, loss, cost etc) Such results are known as pay offs. The average pay off / per play is termed the value of game. A game whose value is zero is called a fair game.

Strategy:- is a plan of action conceived & carefully executed by each party to the game. It involves list of action that player will take for every outcome (pay off) that might arise. The rules governing the choices are known in advance to the players. Also the outcome resulting from a particular choice is also known to the player in advance and is expressed in terms of numerical values.

Optimal strategy:- The particular strategy by which a player optimises his gains or losses without knowing the competitor's strategies is called optimal strategy.

It can be pure or mixed strategy. If the predetermined plan of action (based on which the game are played) doesn't change during the game it is decision rules which is always used by the player to select the particular course of action. It known as pure strategy. If the plan of action which is changed while the game is in progress,

Game Theory

It compares game and business and explore similarities.

1. Features of game

- minimum two players or more
- competition
- having own game plans
- attack / counter attack
- one wins on the cost of others
- I win not only because I played well but your bad game also an addition in my win

2. Features of business.

- two or more rivals
- cutthroat competition
- having own strategies.
- attack / counter attack
- here also one takes advantage on the cost of other disadvantages same here

Subject matter of Game Theory

1. Pure strategies

- single strategies always same.
- no change
- Saddle point

2. Mixed Strategies.

- various strategies change occurs. time to time as per the demand of circumstances.
- Algebraic method
- Arithmetic method
- Matrix method
- Graphical method

Principles of Dominance

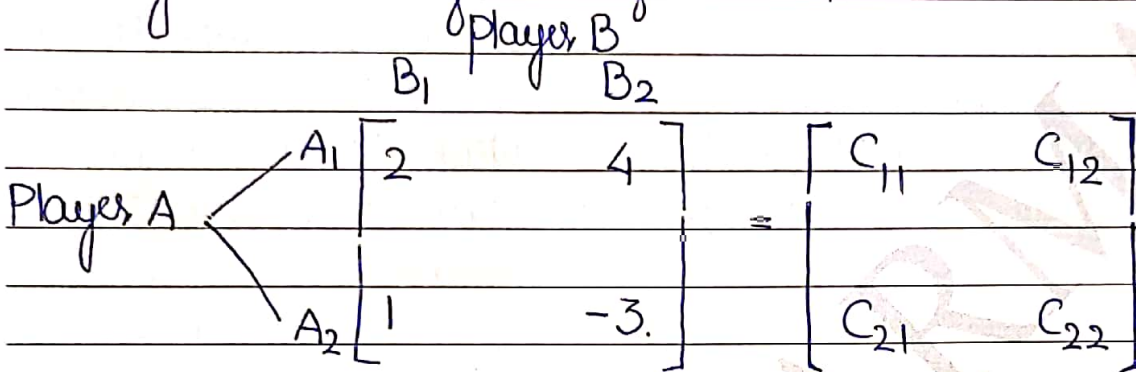
Pure Strategies.

Representation of Game

Representation of Game will be in matrix.
i.e. in row and column (rxc)

Player B having strategies B_1 & B_2

Player A having strategies A_1 & A_2 .



$$r = 2 \quad c = 2$$

$$r \times c \\ = 2 \times 2 \\ = 4$$

If the digits are in positive it means X has won but if negative it means Y is a winner. The detail explanation of pay off matrix are given below.

If A play strategy A_1 & B play strategy B_1
A wins 2 points

If A play strategy A_2 & B play strategy B_1
A wins 1 point

If A play strategy A_1 & B play strategy B_2
A wins 4 points

If A play strategy A_2 & B play strategy B_2
B wins & A loose 3 points

When both players are guessing as to which course of action to be selected on a particular occasion with some fixed profitability. Thus, there is a probabilistic situation and objective of the players is to maximise expected gains or to minimise expected losses by making a choice among pure strategies with fixed probabilities.

Value of the game:- and fair game:-

The maximum guaranteed expected outcome per play when players follow their optimal strategy is called the value of game. If value of game = 0. zero we consider it as a fair game. When the best strategies of both players are determined and value of the game is determined we say the game is solved or the solution of the game is obtained.

Maxmin and Minmax -

The max value of the minimum pay offs in each row. It known as maxmin.

The minimum value of the maximum pay offs in each column. It known as minmax.

Saddle point:- The game value is called S.P in which each player has a pure strategy. The S.P is the lowest numerical value in a row and the largest numerical value in a column, which are equal to each other.

The S.P is the position in the game matrix corresponding to the minimum in its row & max in its column.

Ques- A company management and the labour union are negotiating a 3 year settlement. Each of these has 4 strategies.

1. Hard & aggressive bargaining
2. Reasoning & logical approach
3. Legalistic approach
4. Conciliatory approach

The costs of the company are given for every pair of strategy choice

Union strategies	Company strategies			
	I	II	III	IV
I	20	15	12	35
II	25	14	8	10
III	40	2	10	5
IV	-5	4	11	0

What strategy will the two side adopt?
Also determine the value of game

Solution:-

Union strategies	Company strategies			
	I	II	III	IV
I	20	15 ^(B)	12 ^{(A)(B)}	35 ^(B)
II	25	14	8 ^(A)	10
III	40 ^(B)	2 ^(A)	10	5
IV	-5 ^(A)	4	11	0'

Case of pure strategy

Maximin = Minimax = value of game 12

Saddle point (I, III)

Ques Find the range of values P & Q will render the entry (2,2) a saddle point for the game

		Player B		
		B ₁	B ₂	B ₃
Player A	A ₁	2	4	5
	A ₂	10	7	Q
	A ₃	4	P	6

Solution - first ignoring the values P & Q in pay off matrix, determine the maximin & minimax values in the usual manner

		B ₁	B ₂	B ₃	
A	A ₁	2 ^(A)	4	5	row - min
	A ₂	10 ^(B)	7 ^{(A)(B)}	Q	column - max.
	A ₃	4 ^(A)	P	6 ^(B)	

Saddle point = (A₂B₂) = A₂B₂
 only when $P \leq 7$
 $Q > 7$

Principle of dominance

We can sometimes reduce the size of the game's pay off matrix by eliminating the course of action which is so inferior to another as never to be used. Such course of action is said to be dominated

by the other.

The concept of dominance is useful for evaluation of 2 person zero sum game where a saddle point doesn't exist

Rules:- following are the rules of dominance we use to reduce the size of pay of matrix.

If all element in a row are less than or equal to corresponding element in another row, then that row is dominated & can be deleted from the matrix.

Rule 2:- Similarly if all element in a column are greater than or equal to the corresponding element in another column, then that column is dominated & can be deleted from matrix.

Ques For the following game find optimal strategy of A & B and value of game using principle of dominance

		Player B			
		B ₁	B ₂	B ₃	B ₄
Player A	A ₁	7	6	8	9
	A ₂	-4	-3	9	10
	A ₃	3	0	4	2
	A ₄	10	5	-2	0

A₃ will be omitted

	B ₁	B ₂	B ₃	B ₄
A ₁	7	6	8	9
A ₂	-4	-3	9	10
A ₄	10	5	-2	0

Omitting 4th column i.e B₄ coz B₄ will be omitted (coz larger column will be omitted)

	B ₁	B ₂	B ₃
A ₁	7	6 (A) (B)	8
A ₂	-4 (A)	-3	9 (B)
A ₃	10 (B)	5	-2 (A)

minimum of a row = A

maximum of a column = B

Value of Game = 6 Ans

Ques

Player B

	I	II	III	IV	V
I	2	4	3	8	4
Player A II	5	6	3	7	8
III	6	7	9	8	7
IV	4	2	8	4	3

Using dominance property, obtain the optimal solution / strategy for both the players and determine the value of the game.

row wise row IV omitted by III

	I	II	III	IV	V
I	2	4	3	8	4
II	5	6	3	7	8
III	6	7	9	8	7

IV again row wise I will be omitted by III

	I	II	III	IV	V
II	5	6	3	7	8
III	6	7	9	8	7

column II will be omitted by column I

	I	III	IV	V
II	5	3	7	8
III	6	9	8	7

column IV will be omitted by I

	I	III	V
II	5	3	8
III	6	9	7

column V will be omitted by I

	I	III
II	5	3 ^(A)
III	6 ^{(A)(B)}	9 ^(B)

Value of Game = 6